APPROXIMATE CALCULATION OF THE EVAPORATION OF WATER FROM SOIL WITH A DRY LAYER

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The formation and the motion of water vapor in a layer of dry soil on the surface of a natural ground is investigated. The processes of desorption and sorption, occurring in the presence of temperature change inside a dry soil layer, are regarded as sources of the vapors (positive or negative).

The diffusion equation is solved taking into consideration the vapor sources acting inside the dry soil layer,and the temporal (diurnal) behavior of evaporation of water from soil with a dry layer is explained. We note that the evaporation of water from soil was investigated, for example, in [1, 2]; however, the formation and adsorption of water vapors inside the dry soil was not considered.

In dry soil, only the diffusion mechanism of migration of the vapor form of moisture is possible [3]. The formation and motion of vapor in a dry soil layer in the process of evaporation of water must be described by the equation

$$\frac{\partial q}{\partial t} = D\frac{\partial^2 q}{\partial z^2} + g(z, t) \qquad (0 \leqslant t \leqslant T, \ 0 \leqslant z \leqslant h) \tag{1}$$

with the boundary conditions

$$q_{z=0} = q_0, \qquad q_{z=h} = q_*(h, t) \tag{2}$$

Here q is the density (parameter of state) of the vapor in the pores of the soil, t is time, z is the coordinate measured from the surface of the soil (vertically downward), D is the effective coefficient of diffusion of vapor into the soil determined from G. Penman's formula [4], g(z, t) is the specific intensity of the internal vapor sources, T is the diurnal period, h is the thickness of the dried soil layer, q_0 is the density of vapor in the surface air, and $q_*(h, t)$ is the density of saturated vapor determined by the soil temperature at the level $z = h$.

The validity of conditions (2) is checked experimentally: in the presence of a dried layer of even small thickness on the surface of the soil the vapor density at the soil-air boundary is found [5] to be approximately the same as in the surface air. A moist soil lies under the dry layer. As usually assumed [6], the saturated vapors are located in the pores of the soil; therefore the second boundary condition in (2) follows from the condition of continuity of the vapor density fields at the lower edge of the dry soil. Below,the evaporation is investigated under the assumption of steady-state diffusion, and therefore the initial condition of the problem is not formulated.

We shall assume that the processes of desorption and sorption equalize the gradient of the partial vapor densities, i.e., a distribution of the vapor density q is established according to the law

$$q^\circ(z, t) = \frac{q_*(h, t) - q_0}{h} z + q_0 \tag{3}$$

Observations permit one to assume that the actual distribution of vapors over the profile of the dry soil layer at any instant of time differs little from (3).

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The dependence of the vapor density in the soil pores on the amount of moisture absorbed by the soil is usually determined by empirical formulas [7, 8]. In the range of vapor density possible in soil pores in field conditions the relation between the specific absorbed moisture $a(z, t)$, expressed in grams per gram of dry soil, and the vapor density $q^\circ(z, t)$ in equilibrium with it can be expressed in the first approximation in the form

$$a(z, t) = a_0 + C_1 q^\circ(z, t)\,\psi(z, t) \tag{4}$$

Here a_0 and C_1 are constants depending on the absorbing capability of the soil; $\psi(z, t)$ is a factor that can be approximately written in the form

$$\psi(z, t) = 1 - \alpha_1\vartheta + \alpha_2\vartheta^2 \tag{5}$$

Here α_1 and α_2 are (constant) coefficients and ϑ is the temperature which is a function of z and t. The small deficit, at which the processes of absorption and desorption occur, can be neglected, and it can be assumed that at any time the absorbed moisture is in dynamic equilibrium with the vapor distributed according to (3). In the presence of a temperature variation in the dry soil layer, formation or absorption of moisture occurs, leading to a change of the absorbed moisture content in the soil, and, therefore, the specific intensity of the internal sources can be analytically expressed by means of (4). The mass of vapor getting formed (or absorbed) per unit volume of dry soil per unit time must be equal to the change in the total moisture of the soil (mg/cm^3) per unit time, i.e.,

$$g(z, t) = -\partial m/\partial t = -\rho\,\partial a/\partial t \tag{6}$$

Here ρ is the density of the soil, and a is the specific absorbed moisture given by formula (4) under the assumption that the equilibrium vapors are distributed according to (3).

If we introduce the new variable $u = q - q^\circ$, then under the assumption (6), Eq. (1) together with conditions (2) reduces to an equation with zero boundary conditions of the form

$$\frac{\partial u}{\partial t} = D\frac{\partial^2 u}{\partial z^2} - f(z, t) \tag{7}$$

Here

$$f(z, t) = (C\psi + 1)\frac{\partial q^\circ}{\partial t} + Cq^\circ\frac{\partial \psi}{\partial t}, \qquad C = \rho C_1 \tag{8}$$

As seen from (4)-(6) the specific intensity of the internal sources depends on the variation of the soil temperature. J. Carson [9] has assumed that the regular diurnal behavior of temperature at the surfaces of the soil corresponding to day and night alteration is approximated quite well by the first two harmonics of the Fourier series. However, the relatively small decrease of the temperature in the second half of the night can be neglected, and it can be assumed that the temperature changes according to the sine law from sunrise to midday, and later it does not change till the completion of the diurnal cycle. The proposed method of approximating the diurnal behavior of the temperature at the soil surface, which considerably simplifies the solution of the problem under investigation, is justified in practice at high and middle latitudes during summer months. If the temperature at the surface of the soil changes according to the sine law, then, as is well known [10], a damping thermal wave propagates which can be represented by a semiempirical formula of the form

$$\vartheta(z, t) = \vartheta_0 + (\langle\vartheta\rangle - \vartheta_0)\,e^{-jz} + Ae^{-kz}\sin(\omega t - kz - \varphi_0) \tag{9}$$

Here ϑ_0 is the temperature at a depth to which the thermal wave of diurnal period does not penetrate, $\langle\vartheta\rangle$ is the mean temperature at the surface of the soil, A is the amplitude of the temperature variation at the soil surface, $\omega = 2\pi/T$ is the frequency, j and k are (constant) coefficients, and φ_0 is the phase depending on the choice of the time origin.

In the solution of the problem the following expression is used for the saturation vapor density

$$q_*(z, t) = \beta_0\,[1 + \beta_1\vartheta(z, t) + \beta_2\vartheta^2(z, t)] \tag{10}$$

Here β_0, β_1, and β_2 are coefficients. In the range of temperatures possible at the level $z = h$ in field conditions formula (10) will ensure an accuracy which is adequate for applications, if we take

$$\beta_0 = 5.75 \cdot 10^{-6}\ \mathrm{g/cm^3},\ \beta_1 = 0.0104,\ \mathrm{deg}^{-1},\ \beta_2 = 0.0045\ \mathrm{deg}^{-2}$$

From formulas (9) and (10) we obtain

$$q_*(h, t) = \langle q_h \rangle + Q_1 \sin(\omega t - kh) + Q_2 \sin^2(\omega t - kh) \tag{11}$$

Here $\langle q_h \rangle$, Q_1, and Q_2 are coefficients which do not depend on time.

Under the assumption (3) with (4), (9), and (10) taken into consideration the function (8) reduces to the form

$$\begin{aligned} f(z, t) = {} & (N_0 + N_1 e^{-jz} + N_2 e^{-2jz} + N_3 e^{-2kz}) \frac{z}{h} \frac{\partial q_*(h, t)}{\partial t} \\ & + \frac{e^{-kz}}{h} (N_4 + N_5 e^{-jz}) \left\{ \frac{\partial q_*(h, t)}{\partial t} z \sin(\omega t - kh) + [(q_* - q_0) z + q_0 h]\, \omega \cos(\omega t - kh) \right\} \\ & + \frac{N_3}{h} e^{-2kz} \left\{ 2\omega [(q_* - q_0) z + q_0 h] \sin 2(\omega t - kh) - \frac{\partial q_*(h, t)}{\partial t} z \cos 2(\omega t - kh) \right\} \end{aligned} \tag{12}$$

Here N_i are coefficients which do not depend on the variables. If the constants in formula (4) are determined and the temperature behavior is approximated in form (9) on the basis of observational data, then N_i will be known constants.

If the function (12) is expanded in Fourier sine series in the interval (0, h), Eq. (7) reduces to the form

$$\frac{\partial u}{\partial t} = D \frac{\partial^2 u}{\partial z^2} - \sum_{n=1}^{\infty} f_n(t) \sin \frac{n\pi z}{h} \tag{13}$$

The coefficients $f_n(t)$ of the Fourier series, into which (12) is expanded, represent fourth-order trigonometric polynomials ($\mu = 1$-4), in which only the coefficients depend on n. If the solution of Eq. (13) is sought in the form

$$u = \sum_{n=1}^{\infty} \Phi_n(t) \sin \frac{n\pi z}{h}$$

the zero boundary conditions are automatically satisfied, and in the case of the problem under investigation a dimensionless group of the form

$$\gamma = {}^1/_2\, n^2 \pi D T / \mu h^2 \tag{14}$$

appears in the function $\Phi_n(t)$.

The values of γ in the range of values of h of practical interest are much larger than unity. Actually, from the results of experiments [10] the coefficient D is of the order of 0.1 cm²/sec. In the trigonometric polynomial, which determines $f_n(t)$, the terms with $\mu = 1$ and 2 will be insignificant in practice. If we put $\mu = 2$, $n = 1$ and $T = 72{,}000$ sec, which corresponds to the approximation of the diurnal behavior of the soil temperature used here, the dimensionless group (14) will be more than 100 for h up to 10 cm. The formation and absorption of vapor, caused by the change in the temperature wave of the diurnal period, will be insignificant at a depth of more than 10 cm. The nature of evaporation is mainly affected by the processes of desorption and sorption occurring in the surface layer of the soil in which the inequality $\gamma \gg 1$ holds. If we consider that $\gamma \gg 1$ and make the corresponding simplications, the solution of Eq. (1) in the regime of steady-state diffusion is obtained finally in the form

$$q(z, t) = q_0 + \frac{q_*(h, t) - q_0}{h} z - \frac{h^2}{\pi^2 D} \sum_{n=1}^{\infty} \frac{1}{h^2} f_n(t) \sin \frac{n\pi z}{h} \tag{15}$$

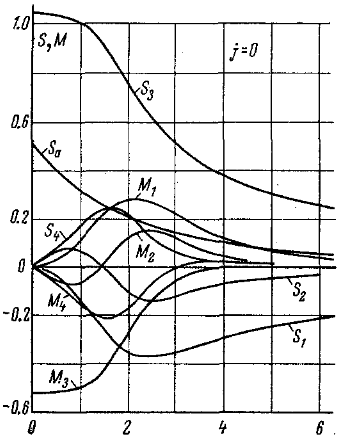

Fig. 1

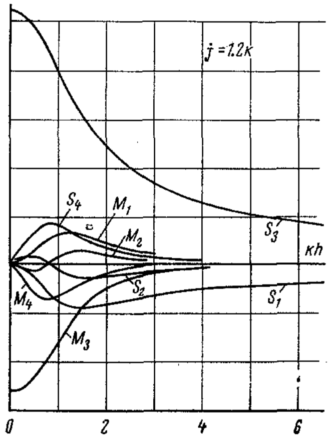

Fig. 2

It must be noted that the first condition in (2) and, hence, formula (15) are valid in the case where there is a dry layer at the surface of the soil, whose thickness is not very small.

Obviously, evaporation from a unit area of soil surface per unit time must be numerically equal to the density of the diffusion flux of vapor E(0, t) through the soil surface. It follows from formula (15) that

$$E(0, t) = \left| -D\frac{\partial q}{\partial z} \right|_{z=0} = D\frac{q_*(h, t) - q_0}{h} - \frac{h}{\pi}\sum_{n=1}^{\infty}\frac{f_n(t)}{n} \tag{16}$$

According to formula (16) the evaporation of water from a soil without vegetation cover must be treated as the sum (superposition) of fluxes of vapors, forming under the dry soil layer (first term of the formula), and the vapor forming (or getting absorbed) during the change of temperature inside the dry soil layer (second term of the formula). In the case of heating of the soil the vapor forming as a result of desorption inside the dry soil layer is added to the vapor flux diffusing across the dry layer from the depth of the soil. At the stage of temperature decrease in the dry soil layer, an absorption of the vapors occurs from the flux coming from the zone of moist soil. In the presence of vegetation cover on the surface of the soil, vapors forming due to transpiration are added to the vapor flux determined by formula (16).

The second term of formula (16) takes into account the effect of the processes of desorption and sorption occurring inside the dry soil on the nature of evaporation. As mentioned above, $f_n(t)$ represents a trigonometric polynomial in which only the coefficients depend on n; therefore the second term in (16) is also a trigonometric polymial whose coefficients contain dimensionless factors determined by the series

$$S_0(k) = \sum_{n=1}^{\infty}\frac{B_{0n}(k)}{n}, \quad S_i(j, k) = \sum_{\substack{n=1 \\ (i=1,\,2,\,3,\,4)}}^{\infty}\frac{B_{in}(j, k)}{n}, \quad M_i(j, k) = \sum_{n=1}^{\infty}(-1)^n\frac{B_{in}(j, k)}{n} \tag{17}$$

$$\begin{aligned}
B_{0n}(k) &= \frac{2\pi n\,[2kh - (-1)^n e^{-kh}(2kh + k^2h^2 + n^2\pi^2)]}{(k^2h^2 + \pi^2n^2)^2} \\
B_{1n}(j, k) &= \frac{(j + k)\,h}{(j + k)^2h^2 + (kh + \pi n)^2} - \frac{(j + k)\,h}{(j + k)^2h^2 + (\pi n - kh)^2} \\
B_{2n}(j, k) &= \frac{(j + k)^2h^2 - (\pi n + kh)^2}{[(j + k)^2h^2 + (\pi n + kh)^2]^2} - \frac{(j + k)^2h^2 - (\pi n - kh)^2}{[(j + k)^2h^2 + (\pi n - kh)^2]^2} \\
B_{3n}(j, k) &= \frac{\pi n + kh}{(j + k)^2h^2 + (\pi n + kh)^2} + \frac{\pi n - kh}{(j + k)^2h^2 + (\pi n - kh)^2} \\
B_{4n}(j, k) &= \frac{2(j + k)\,h\,(\pi n + kh)}{[(j + k)^2h^2 + (\pi n + kh)^2]^2} + \frac{2(j + k)\,h\,(\pi n - kh)}{[(j + k)^2h^2 + (\pi n - kh)^2]^2}
\end{aligned} \tag{18}$$

The dimensionless factors (coefficients S_i and M_i) depending only on the dimensionless parameter kh (and jh) can be found beforehand with the use of formulas (18) and (19). The coefficients S_i and M_i are shown in the form of graphs in Figs. 1 and 2 as functions of kh computed on a computer (with an accuracy up to n = 50) for j/k = 0 (Fig. 1) and j/k = 1.2 (Fig. 2). The compilation of the program and the computations were

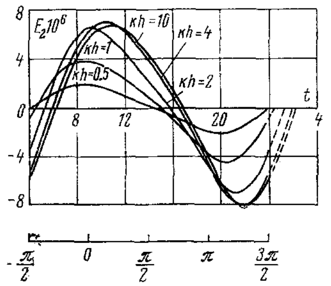

Fig. 3

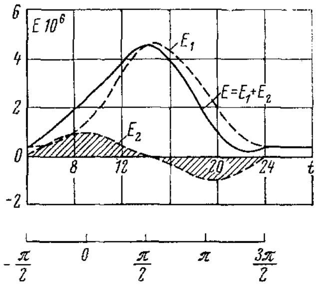

Fig. 4

done by G. A. Gerasimov, When the dimensionless coefficients (S_i and M_i) have been computed, the second term in formula (16) does not contain the series and will be represented through exponential functions.

The coefficients of the trigonometric polynomial representing the second term of formula (16) were calculated on a computer separately in a wide range of variation of kh and for the values of the remaining parameters possible in practice in field conditions. The results of these computations show that certain harmonics functions, in which the amplitude values are relatively small, can be neglected; then we obtain

$$
\begin{aligned}
\frac{T}{2\pi}\sum_{n=1}^{\infty}\frac{f_n(t)}{n} \approx {} & \{(\langle q_k\rangle + {}^1\!/\!_4 Q_2)(P_1+P_2) - q_0P_2 + {}^1\!/\!_2 Q_1 N_3 e^{-kh}\,[M_1(0,2k)+M_3(0,2k)]\} \\
& \times e^{-kh}\sin(\omega t - kh) + \{Q_1\eta - [(\langle q_h\rangle + {}^3\!/\!_4 Q_2)(P_3+P_4) - q_0P_4]\,e^{-kh} \\
& + {}^1\!/\!_2 Q_1 N_3\,[M_3(0,2k) + M_4(0,2k)]\,e^{-2kh}\}\cos(\omega t - kh) - (Q'G_2 + q_0G_1)\sin\omega t \\
& + (Q'G_4 + q_0G_3)\cos\omega t + Q_1\,[G_4\sin(2\omega t - kh) + G_2\cos(2\omega t - kh)] \\
& + \{Q_2\eta - Q_1\,[P_3+P_4]\,e^{-kh} - 2N_3\,[Q'M_4(0,2k) + (\langle q_h\rangle - {}^1\!/\!_2 Q_2)\,M_3(0,2k)]\,e^{-2kh}\} \\
& \times \sin 2(\omega t - kh) + 2N_3\,[q_0S_3(0,2k) + Q'S_4(0,2k)]\sin 2\omega t \\
& + 2N_3\,[q_0S_1(0,2k) + Q'S_2(0,2k)]\cos 2\omega t
\end{aligned}
\tag{19}
$$

Here

$$
\eta = -\frac{2N_0}{\pi}\sum_{n=1}^{\infty}(-1)^n\frac{1}{n^2} + N_1S_0(j) + N_2S_0(2j) + N_3S_0(2k)
$$

$$
Q' = \langle q_h\rangle + {}^1\!/\!_2 Q_2 - q_0, \qquad G_i = N_4S_i(0,k) + N_5e^{-jh}S_i(j,k)
$$

$$
P_i = N_4M_i(0,k) + N_5e^{-jh}M_i(j,k) \qquad (i=1,2,3,4)
$$

The values of j and k, for which S_i and M_i must be determined from Fig. 1 and Fig. 2 are written in parentheses beside the coefficients S_i and M_i. For example, S(0, 2k) is the coefficient which is determined from Fig. 1 for the value of the argument 2kh.

It follows from formulas (16) and (19) that evaporation from a soil with dry layer depends on sorption and thermal properties of the soil, on the moisture in the surface air, and principally on the behavior of the temperature and the thickness of the dry soil layer.

The effect of the processes of desorption and sorption on the nature of evaporation is illustrated in Fig. 3 as a function of the thickness of the dry soil layer; the second term of formula (16), i.e., the flux of vapors forming in the presence of time variations of E, $g \cdot cm^{-2} \cdot sec^{-1}$, is shown in the form of a graph for different values of the parameter kh; plotted along the t axis is the time (in hours) reckoned from the minimum in the curve of the temperature at the surface of the soil. The curves in Fig. 3 are constructed from the results of computations done on a computer taking T = 72,000 sec, $\langle\vartheta\rangle = 28°C$, $\vartheta_0 = 20°C$, A = 12°C, $\rho = 1.4$ g/cm^3, $q_0 = 10 \cdot 10^{-6}$ g/cm^3, $C_1 = 4.85 \cdot 10^3$ cm^3/g, $\alpha_1 = 4.3 \cdot 10^{-2}$ deg^{-1}, $\alpha_2 = 4.7 \cdot 10^{-4}$ deg^{-1}.

The coefficients (C_1, α_1 and α_2) in formulas (4) and (5), used in the computation, were found from the graph of the family of sorption isotherms of a loamy layer, constructed from the data of field experiments. As is evident, the graph of the second term of (16) represents a curve which is similar to sinusoid in form, displaced in phase depending on the parameter kh. The sinusoidal form of the curve is due to the fact that a periodic variation of the distribution of adsorbed moisture occurs as a result of the processes of desorption and sorption in the presence of a regular variation of temperature in the dry soil, which appears in the form of a hydrologic cycle in the course of the day. It is evident from Fig. 3 that as the thickness of the dry soil layer (parameter kh) increases, the amplitude value of the curve characterizing the effect of the hydrologic cycle on the nature of evaporation increases at first almost proportional to the thickness h, and at large values h approaches a constant limit. The regularity with which the degree of the effect of the hydrologic cycle on the nature of evaporation changes depending on h is easily explained by the weakening of the processes of desorption and sorption in the depth of the soil. The diurnal variation of temperature leads to the formation and absorption of vapors mainly in the surface layer of the soil. In view of this the amount of moisture, which participates in the hydrologic cycle and affects evaporation, ceases to depend on h on increasing the thickness of the dry soil layer to an appreciable value.

For sufficiently small values of h the second term of (16) will be apparently smaller than the first. Hence, according to (16), for a small thickness of the dry soil layer the evaporation is mainly determined by the diffusion flux (across the dry layer) of vapors formed in the depth of the soil (in the zone of evaporation).

The behavior of evaporation, computed from formula (16) for $h = 1.25$ cm ($kh = 0.25$), $j/k = 1.2$ and for the values of the remaining parameters given above is shown in Fig. 4 in the form of a graph. Farrell et al. [12] assume that due to the partial penetration of atmospheric turbulence into the soil the transport of vapors in the soil is increased. In the case of the example shown in Fig. 4 the effect of atmospheric turbulence on the evaporation is taken into consideration through the introduction of the nondimensional factor $f' = 1.4$ in the formula for the effective coefficient of diffusion of vapors in the soil. The shaded area of Fig. 4 characterizes the amount of moisture taking part in the hydrologic cycle in the presence of periodic (diurnal) variation of the soil temperature. The components of evaporation are also shown in Fig. 4; the curve E, given by the first term of formula (16), shows the fraction of evaporation due to diffusion of vapors through the dry soil layer, which are formed in the zone of moist soil; the curve E_2, determined by the second term of (16), shows the fraction of evaporation caused by the process of desorption and sorption occurring inside the dry soil layer. In the example shown in Fig. 4 the second component of evaporation is comparable to the first, i.e., the hydrologic cycle occurring in dry soil of thickness 1.25 cm affects the evaporation significantly. Since the second term of (16) increases with the parameter kh, while the first term decreases, for an appreciable thickness of the dry soil layer the behavior of evaporation must depend mainly on the processes of desorption and sorption occurring inside this soil layer.

The behavior of evaporation described by formula (16) must be interpreted as typical. Actually the behavior of evaporation corresponds to the diurnal behavior of the temperature approximated in form (9), in which random deviations caused by weather conditions of nonperiodic character have been smoothed out.

The proposed simplified method of representation of the diurnal behavior of temperature, which has been used in the present work, leads to the result that the behavior of evaporation in the stage of temperature decrease is described in a form with deformation along the t axis. For eliminating this small error caused by this simplification the temperature behavior must be approximated by the first two to three harmonics of the Fourier series.

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